ANSWERS XIV

CHEMISTRY

1a	2.a	3.c	4.c	5.b	6.d	7.c	8.a	9.d	10.d	11.a	12.c	13.d
14.a	15.c	16.a	17.c	18.b	19.b	20.a	21.a	22.b	23.c	24.c	25.b	26.b
27.a	28.a	29.a	30.a									

PHYSICS

1.c	2.d	3.b	4.a	5.b	6.a	7.d	8.a	9.a	10.d	11.a	12.a	13.d
14.a	15.c	16.a	17.d	18.a	19.b	20.b	21.b	22.d	23.a	24.b	25.c	26.d
27.c	28.a	29.d	30.d									

MATHEMATICS

1.b	2.a	3.d	4.c	5.a	6.d	7.c	8.b	9.a	10.a	11.b	12.c	13.c
14.d	15.a	16.d	17.b	18.c	19.d	20.a	21.b	22.a	23.a	24.d	25.c	26.a
27.b	28.a	29.b	30.d									

HINTS AND EXPLANATIONS XIV

CHEMISTRY

Sol.1

Nitrogen can be estimated by Kjeldahl's and halogens are estimated by Carius method.

Sol.2

When subjected to high pressure NaCl type structure will transform to 8 : 8 coordination

Sol.3

$Ni(CO)_4$ and $[Ni(CN)_4]^{2-}$ are diamagnetic and $[NiCl_4]^{2-}$ is paramagnetic.

Sol.4

Apply Boyle's law,

$P_1 V_1 = P_2V_2$ (T is constant)

$P \times 100 = P_2 \times 50$

$P_2 = 2P$

Apply charle's law, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$; $\frac{50}{400} = \frac{75}{T_2} = \frac{75 \times 400}{50} = 600K$

Sol.5

Al cannot be extracted by electrolysis of fusedaluminium chloride because it is a covalent compound and is a poor conductor of electricity.

Sol.6

On increasing carbon contents in steel ductility decreases but tensile strength increases.

Sol.7

Ammonium phosphate on heating gives ortho phosphoric acid (H_3PO_4) and ammonia; ortho phosphoric acid on heating at 873K forms metaphophoric acid (HPO_3)

Sol.8

$5H_2O_2 + 2KMnO_4 + 3H_2SO_4 \rightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5O_2$

5 x 34g 2 x 158 = 316g 5 x 32 = 160g

34 g of H_2O_2 will react with 316/5 = 63.2 g of $KMnO_4$ to form 160/5 = 32g of O_2 Here H_2O_2 is a limiting reagent.

Sol.9

$CH_3CH_2CH_2CH_2OH$ on heating with Al_2O_3 undergoes dehydration to form an alkene and not an aldehyde.

Sol.10

$C_2H_5Br + AgCN \rightarrow C_2H_5NC + Ag\,Br$;

C_2H_5NC on hydrolysis forms $C_2H_5NH_2 + HCOOH$

Sol.11

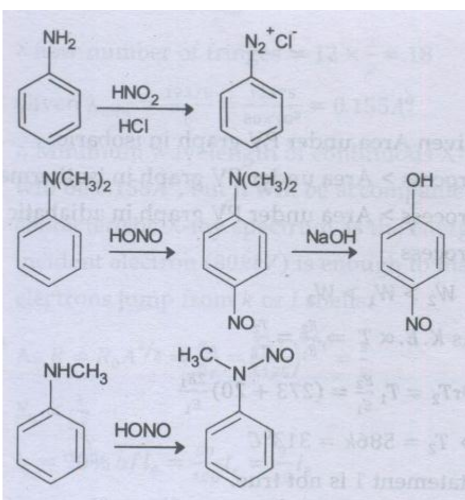
Lead (II) is more stable due to inert pair effect.

Sol.12

$(HCOO)_2Ca + (CH_3COO)_2Ca \rightarrow 2\,CaCO_3 + CH_3COCH_3$

$CH_3COCH_3 + 3Cl_2 + NaOH \rightarrow 2CHCl_3 + CH_3COONa$

Sol.13



Sol.14

$\Delta G = \quad -2.303RT\log K_p$

$= \quad -\ 2.303 \times 8.314 \times 250 \times \log 10^4$

$= \quad -\ 19.14 \times 250 \times 4 = \quad -\ 19.14 \times 1000\ J = \quad -\ 19.14kJ$

Sol.15

H_2O_2 reduces Fe^{3} to Fe^{2+} so act as a reducing agent.

Sol.16

Fitting reaction

Sol.17

$CH_3COONa + BrCH(CH_3)_2 \rightarrow CH_3COOCH(CH_3)_2$

Sol.18

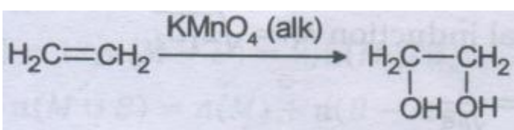
Since the rate of reaction becomes nine times when the conc. Of both the reactants are tripled simultaneously, it is a second order of reaction.

Sol.19

In an invert sugar both glucose and fructose are in pyranose form.

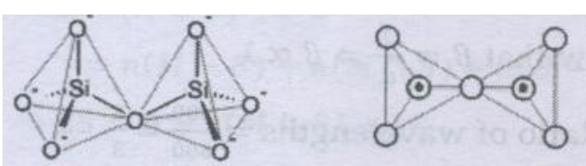
Sol.20

Ethane reacts with $KMnO_4$ to form ethane -1,2-diol (glycol)



Sol.21

Pyro silicates contain $Si_2O_7^{6-}$ ions which are formed by joining two tetrahedral SiO_4^{4-} which share one oxygen atom at one corner



Sol.22

$ZnSO_4 \rightarrow Zn^{2+} \quad SO_4^{2-}$

$1 - 0.4 \quad 0.4 \quad 0.4$ (when 40% dissociated)

Total moles = 1 – 0. 4 + 0. 4 + 0. 4 = 1. 4

Osmotic pressure (π) is : π =i.nRT

= 1.4 x 0.1 x 0.082 x 273 = 3.134

Sol.23

All α-amino acids have L- configuration.

Sol.24

Copper has higher reduction potential than hydrogen so it cannot liberate H_2 gas from an acid.

Sol.25

In sequestration or calgon method of water softening, hard water is treated with sodium polymetaph osphate to form soluble non-interfering Ca & Mg complexes.

Ca^{2} + $(NaPO_3)_n \rightarrow 2Na^+$ + $Na_{n-2}Ca(PO_3)_n$

(from hard water) Calgon (Soluble)

Mg^{2+} + $(NaPO_3)_n \rightarrow 2Na^+$ + Na^+ + $Na_{n-2}Ca(PO_3)_n$

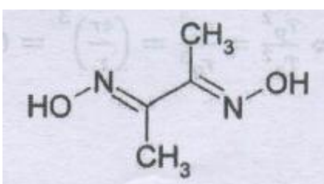
(from hard water) Calgon (Soluble)

Sol.26

The first I.E. of 5d row elements is higher than 3d & 4d elements due to poor shielding of 4f electrons.

Sol.27

The ligand dimethyl glyoxime (DMG) is bidentate



Sol.28

Emulsification the process of dispering one liquid in a second immiscible liquid.

Sol.29

$SnCl_2$ can easily undergo oxidation to form $SnCl_4$; it acts as a reducing agent. $PbCl_4$ – acts as an oxidising agent as it changes to more stable $PbCl_2$

Sol.30

The solution of alkali metals in ammonia are blue in colour due to formation of ammoniated electrons $[e(NH_3)_y]$.

PHYSICS

Sol.1

Least $= \frac{pitch}{number\ of\ divisions} = \frac{0.625}{250} = 2.5 \times 10^{-3} cm$

Sol.2

Given $\sin\theta = \frac{v}{u} = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = 30^0$

Here θ is the angle with direction of width.

∴ The angle with the direction of flow of river

$= 90 + \theta = 90 + 30 = 120^0$

Sol.3

By the principal of conservation of momentum

$Mv = -mv = mv'$

$Mv = mv = mv' \Rightarrow v' = \left(\frac{m+M}{M}\right) v = \left(\frac{m+2m}{m}\right) v = \frac{3m}{m} v$

Or $v' = 3v$

Sol.4

Given Power $= p \times \frac{v}{t} = 20000 \times 1 \times 10^{-6} = .02W$

Sol.5

From the relation $V = \sqrt{3gr}$

$= \sqrt{3 \times 98 \times 1} = 5.4m/s$

Sol.6

As $T^2 \propto r^3 \Rightarrow \frac{{T_p}^2}{{T_q}^2} = \frac{r_p^3}{r_q^3} = \left(\frac{4r}{r}\right)^3 = 64 \Rightarrow \frac{T_p}{T_q} = 8$

$\therefore \frac{2\pi r_p/v_p}{2\pi r_q/v_p} = \frac{r_p v_p}{r_q v_p} = 8$

$\Rightarrow V_q = \frac{8}{4} \times 3v = 6v$

Sol.7

We know that compressibility = $\frac{1}{bulkmodulus}$

$\Rightarrow K = \frac{1}{4\times10^{-5}} = 0.25\times10^{5}atm = 2.533\times10^{9}N/m^{2}$

$P = 100atm = 1.013\times10^{7}N/m^{2}$

$v = 100cm^{3} = 100\times(10^{-2}m)^{3} = 10^{-4}m^{3}$

$K = \frac{P}{\Delta V/V} = \frac{PV}{\Delta V} \Rightarrow \Delta V = \frac{PV}{K} = \frac{1.013\times10^{7}\times10^{-4}}{2.533\times10^{9}}$

$\therefore \Delta V = 0.4\times10^{-6}m^{3} = 0.4cm$

Sol.8

As we know work done = Area under PV graph

Give area under PV graph in isobaric

Process > Area under PV graph in isothermal

Process > Area under PV graph in adiabatic process

$\therefore W_2 > W_1 > W_3$

Sol.9

As $K.E. \propto T \ \Rightarrow \frac{E_2}{E_1} = \frac{T_2}{T_1}$

Or $T_2 = T_1\frac{E_2}{E_1} = (273+20)\frac{2E_1}{E_1}$

$\Rightarrow T_2 = 586K = 313^{0}C$

Sol.10

Statement 1 is not true .

Sol.11

$\frac{1}{K} = \frac{1}{K\prime} + \frac{1}{K\prime}$ Here K'= new spring constant

$= \frac{2}{K'} \Rightarrow K' = 2K$

Sol.12

The introduction of dielectric slab does not change the direction of electric field.

Sol.13

Given $\frac{P}{Q} = \frac{l_1}{l_2} = \frac{2}{3}$ $\Rightarrow \frac{5}{Q} = \frac{2}{3}$ or

$Q = \frac{15}{2} = 7.5\Omega$

Sol.14

It will be diamagnetic .

Sol.15

As$R = \frac{I_g^G}{I - I_g} = \frac{(1\times10^{-3})(20)}{1-(1\times10^{-3})} = 0.02\Omega$

Sol.16

The alternating currents are always transmitted at high voltage and low current to distant places

Sol.17

Mutual induction $M = \sqrt{L_1 L_2}$

Sol.18

As $v = \frac{1}{\sqrt{\mu \in}}$

Sol.19

As $\mu = \frac{d_{real}}{d_{apparent}} = \frac{2d}{0.6\times 2d} = 1.66$

Sol.20

The direction of emergence will remain same.

Sol.21

We know that $\beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \lambda$ Given Ratio of wavelengths$= \frac{400}{600} = \frac{2}{3}$ $\Rightarrow$no. of fringes increase by $3/_2$times. $\therefore$ new number of fringes $= 12 \times \frac{3}{2} = 18$

Sol.22

Given $\lambda_{min} = \frac{12378}{80\times10^3} = 0.155A^0$

$\therefore$ Minimum wavelength of continuous X-ray will be $0.155A^0$, but it will be accompanied by characteristic X-ray spectrum as the energy of incident electron ($80keV$) is enough to make electrons jump from $kor\ l$ shells.

Sol.23

As $R = R_0 A^{1/3} \Rightarrow \frac{R_A}{R_r} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}$

Sol.24

$E_n \propto \frac{1}{n_2}$

Sol.25

$I_c = 90\%\ of\ I_e = \frac{90}{100} I_e = \frac{9}{10} I_e$

$\Rightarrow I_e = \frac{10}{9} I_c = \frac{10}{9} \times 10 = 11.1mA$

$I_0 = I_e - I_c = 11.1 - 10 = 0.1mA$

Sol.26

$\frac{N_{x_1}(t)}{N_{x_2}(t)} = \frac{1}{e}\ or \frac{N_0 e^{-10\lambda t}}{N_0 e^{-\lambda t}} = \frac{1}{e}$

As both have same number of nuclei (N_0) initially

$\Rightarrow e = \frac{e^{-\lambda t}}{e^{-10\lambda t}}\ or\ e = e^{9\lambda t}$

$\Rightarrow 9\lambda t = 1\ or\ t = \frac{1}{9\lambda}$

Sol.27

Speed of sound in ideal gas is $V = \sqrt{\frac{\gamma RT}{M}} \Rightarrow V \propto \sqrt{\frac{\gamma}{M}}$

$\therefore \frac{V_{N_2}}{V_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}}, \frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3}\left(\frac{4}{28}\right)} = \sqrt{\frac{3}{5}}$

$\therefore \gamma_{N_2} = \frac{7}{5}\ (Diatomic)$

$\gamma_{He} = \frac{5}{3} (Monoatomic)$

Sol.28

$R_1 = -R, R_2 = +R, \mu_g = 1.5$ and $\mu_m = 1.75$

$\therefore \frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_1}\right) = \left(\frac{1.5}{1.75} - 1\right)\left(\frac{1}{-R} - \frac{1}{R}\right)\ \frac{1}{f} = \frac{1}{3.5R} 4 \Rightarrow f = +3.5R$

Thus it will behave like a convergent lens of focal length 3.5R

Sol.29

Since the capacitor plates are directly connected to the battery, it will take no time in charging

Sol.30

In beta decay, atomic number increases by 1 while the mass number remaining the same. Thus the following equation may be possible

$${}^{64}_{29}Cu \rightarrow {}^{64}_{29}Zn + {}_{-1}e^{0}$$

MATHEMATICS

Sol.1

Let m denotes the set of student who have taken mathematics, B the set of students who have taken biology.

$n(M) = 12, n(M - B) = 8, n(M \cup B) = 25$ Now $n(M \cup B) = n(M) + n(B - M)$

$25 = 12 + n(B - M)$

$n(B - M) = 13$ Also $n(M \cup B) = n(M - B) + n(M \cap B) + n(B - M)$ $\Rightarrow 25 = 8 + n(M \cap B) + 13$

$n(M \cap B) = 4$

$\therefore$ number of students who have taken both mathematics and biology is 4.

Sol.2

Let $f(x) = \sin\left(\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$

For f to be defined, $\frac{\sqrt{4-x^2}}{1-x} > 0$

$\therefore \sqrt{4 - x^2 > 0}$ and $1 - x > 0$

$\Rightarrow 4 - x^2 > 0$ and $1 > x$

$\Rightarrow x^2 < 4$ and $x < 1$

$\Rightarrow |x|^2 < 2$ and $x < 1$

$\Rightarrow |x| < 2$ and $x < 1 \Rightarrow -2 < x < 2$ and $x < 1$ $\therefore D_f = (-2, 1)$

Sol.3

Given that

$\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$

$\Rightarrow 2\left(\frac{\cos A}{a} + \frac{\cos C}{c}\right) + \frac{\cos B}{b} = \frac{a^2+b^2}{abc}$

$\Rightarrow 2\left(\frac{c\cos A + a\cos C}{ca}\right) + \frac{\cos B}{b} = \frac{a^2+b^2}{abc}$

$\Rightarrow 2\left(\frac{b}{ca}\right) + \frac{c^2+a^2-b^2}{2cab} = \frac{a^2+b^2}{abc}$

$\Rightarrow 4b^2 + c^2 + a^2 - b^2 = 2?\,(a^2 + b^2)$

$\Rightarrow b^2 + c^2 = a^2$

$\Rightarrow A = 90^0$

Sol.4

$L.H.S. = sin^4\frac{\pi}{8} + sin^4\frac{3\pi}{8} + sin^4\frac{5\pi}{8} + sin^4\frac{7\pi}{8}$

$= sin^4\frac{\pi}{8} + sin^4\frac{3\pi}{8} + sin^4\left(\pi - \frac{3\pi}{8}\right) + sin^4\left(\pi - \frac{\pi}{8}\right)$

$= sin^4\frac{\pi}{8} + sin^4\frac{3\pi}{8} + sin^4\frac{3\pi}{8} + sin^4\frac{\pi}{8}$

$= 2\left[sin^4\frac{\pi}{8} + sin^4\frac{\pi}{8}\right]$

$= 2\left[sin^4\frac{\pi}{8} + sin^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right]$

$= 2\left[sin^4\frac{\pi}{8} + cos^4\frac{\pi}{8}\right]$

$= \frac{1}{2}\left[\left(2sin^2\frac{\pi}{8}\right)^2 + \left(2\,cos^2\frac{\pi}{8}\right)^2\right]$

$= \frac{1}{2}\left[\left(1 - \cos\frac{\pi}{4}\right)^2 + \left(1 + \cos\frac{\pi}{4}\right)^2\right]$

$= \frac{1}{2}\left[1 + cos^2\frac{\pi}{4} - 2\cos\frac{\pi}{4} + 1 + cos^2\,\frac{\pi}{8} + 2\cos\frac{\pi}{4}\right]$

$= \frac{1}{2}\left[2 + 2cos^2\frac{\pi}{4}\right] = 1 + cos^2\frac{\pi}{4} = 1 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 + \frac{1}{2} = \frac{2}{3}$

Sol.5

$$L.H.S. = \left(\frac{1+\sqrt{3}}{-1+\sqrt{3}}\right)^{200} + \left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{200}$$

$$= \left[\frac{i\left(i+\sqrt{3}\right)}{i\left(-i+\sqrt{3}\right)}\right]^{200} + \left[\frac{i\left(i-\sqrt{3}\right)}{i\left(i+\sqrt{3}\right)}\right]^{200}$$

$$= \left(\frac{-1+i\sqrt{3i}}{1+\sqrt{3i}}\right)^{200} + \left(\frac{-1-\sqrt{3i}}{-1+\sqrt{3i}}\right)^{200}$$

$$= \left[\frac{\frac{-1+i\sqrt{3}}{2}}{\frac{-1-i\sqrt{3}}{2}}\right]^{200} + \left[\frac{\frac{-1-i\sqrt{3}}{2}}{\frac{-1+i\sqrt{3}}{2}}\right]^{200}$$

$$= \left(\frac{\omega}{\omega^2}\right)^{200} + \left(\frac{\omega^2}{\omega}\right)^{200}$$

$$= \frac{1}{\omega^{200}} + \omega^{200}$$

$$= \frac{1}{(\omega^3)^{66}\omega^2} + (\omega^3)^{66}.\omega^2$$

$$= \frac{1}{\omega^2} + \omega^2 = \frac{1+\omega^4}{\omega^2} = \frac{1+\omega}{\omega^2} = -\frac{\omega^2}{\omega^2}$$

$$= -1$$

Sol.6

Given $\frac{1}{x+1} - \frac{4}{(2+x)^2} > 0$

$\Rightarrow \frac{(2+x)^2-4(x+1)}{(x+1)(2+x)^2} > 0$

$\Rightarrow \frac{x^2+4x+4-4x-4}{(x+1)(2+x)^2} > 0$

$\Rightarrow \frac{x^2}{(x+1)^2}, \frac{1}{x+1} > 0$

$\Rightarrow \frac{1}{x+1} > 0 \left[\therefore \frac{x^2}{(2+x)^2} > 0\right]$

$\Rightarrow x + 1 > 0$

$\Rightarrow x > -1$

$\Rightarrow x \in (-1, \infty)$

$\therefore$ Solution set is $(-1, \infty)$

Sol.7

Given word is INVOLUTE, Here number of constants = 4 and number of vowels = 4

2 constants can be selected out of 4 constants in ${}^4c_3$ ways and 3 vowels out of 4 vowels can be selected in ${}^4c_3$ ways. ∴ Total number of groups formed = ${}^4c_2 \times {}^4c_3 = 6 \times 4 = 24$

Now each group contains 5 letters which can be arranged themselves in 5! Ways.

∴ Required number of words formed $= 24 \times 5 = 24 \times 120 = 2880$

Sol.8

Consider $\left(\sqrt{2} + 3^{\frac{1}{5}}\right)^{10}$ The only rational terms in the expansion are ${}^{10}c_0\left(2^{\frac{1}{2}}\right)^{10}$ and ${}^{10}c_{10}\left(3^{\frac{1}{5}}\right)^{10}$

i.e. 2^5 and 3^2

i.e. 32 and 9

∴ Required sum = 32 + 9 = 41

Sol.9

Let $S_n = 1 + 3 + 7 + 15 + 31 + \cdots + T_{n-1} + T_n$

$S_n = 1 + 3 + 7 + 15 + \cdots + T_{n-2} + T_{n-1} + T_n$

Subtracting (ii) from (i), we get

$0 = 1 + [2 + 4 + 8 + 16 + \ldots to (n-1) terms]$

$-T_n$

$\Rightarrow T_n = 1 + \frac{2(2^{n-1}-1)}{2-1}$

$\Rightarrow T_n = 1 + 2(2^{n-1} - 1)$

$\Rightarrow T_n = 2^n - 1$

Putting $n = 1, 2, 3, \ldots \ldots \ldots \ldots \ldots, n$ and adding

$S_n = (2 + 2^2 + 2^3 + \cdots + 2^n) - n$

$= \frac{2(2^n-1)}{2-1} - n$

$= 2(2^n - 1) - n$

$S_n = 2^{n+1} - n - 2$

Sol.10

The equation ofAB through $A(7,0)$ and $B(0,-5)$ is $\frac{x}{7}+\frac{y}{5}=1$, Equation of PQ perpendicular to AB is

$\frac{x}{5}+\frac{y}{7}=K$ Or $\frac{x}{5K}+\frac{y}{7K}=1$

Which meets x axis in $P(5K,0)$ and y axis in $Q(0,7K)$ The equation of AQ is

$y-0=\frac{7K-0}{0-7}(x-7)$ Or $y=-K(x-7)$ (i)

The equation of BP is $y+5=\frac{0+5}{5K-0}(x-0)$ Or $y+5=\frac{x}{K}$ (ii)

Multiplying (i) and (ii), we get

$y(y+5)=-x(x-7)$ Or $x^2+y^2-7x+5y=0$ Which is locus of R

Sol.11

Since centre lies on x axis $\therefore$ we take centre as $(h,0)$ Radius of circle = 5

$\therefore$ Equation of circle is

$(x-h)^2+(y-0)^2=(5)^2$

$\Rightarrow (x-h)^2+y^2=25$ (i)

$\because$ It passes through (2, 3)

$\therefore (2-h)^2+9=25$

$\Rightarrow (2-h)^2=16$

$\Rightarrow 2-h=\pm 4$

$\Rightarrow h=-2,6$

Where $h=6,(i)$ be comes

$(x-6)^2+y^2=25$

Or $x^2+y^2-12x+11=0$

Sol.12

Let the equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (i) ∴ It passes through (4, 3)

$\therefore \frac{16}{a^2}+\frac{9}{b^2}=1$ (ii) Also (i) passes through $(-1,4)$ $\therefore \frac{1}{a^2}+\frac{16}{b^2}=1$ (iii)

Multiplying (ii) by 16 and (iii) by 9 and subtracting we get

$\frac{247}{a^2}=7 \Rightarrow \frac{1}{a^2}=\frac{7}{247}$ From (ii), $\frac{112}{247}+\frac{9}{b^2}=1 \Rightarrow \frac{9}{b^2}=1-\frac{112}{247}=\frac{135}{247} \Rightarrow \frac{1}{b^2}=\frac{15}{247}$ Putting value of $\frac{1}{a^2},\frac{1}{b^2}$ in (i), we get

$\frac{7x^2}{247}+\frac{15}{247}y^2=1 \Rightarrow 7x^2+15y^2=247$

Sol.13

Let $y=(1+x)^{\frac{1}{x}}$

$\text{Log } y=\frac{1}{x}\log(1+x) = \frac{1}{x}\left(x-\frac{x^2}{2}+\frac{x^3}{3}\dots\dots\dots\dots\right)$

$\text{Log } y=1-\frac{x}{2}+\frac{x^2}{3}\dots\dots\dots\dots.$

$\Rightarrow y=e^{1-\frac{x}{2}+\frac{x^2}{3}\dots\dots\dots\dots} = e^{1-(\frac{x}{2}-\frac{x^2}{3}+\cdots)} = e.e^{-(\frac{x}{2}-\frac{x^2}{3}\dots\dots\dots)}$

$=e\left[1-\left(\frac{x}{2}-\frac{x^2}{3}+\cdots\right)+\frac{\left(\frac{x}{1}-\frac{x^2}{3}+\cdots\right)^2}{2!}+\cdots\right]$

$=e\left[1-\frac{x}{2}+x^2\left(\frac{1}{3}+\frac{1}{8}\right)+\cdots\right]$

$=e\left[1-\frac{x}{2}+\frac{11}{24}x^2+\cdots\right]$

$\Rightarrow y=e-\frac{ex}{2}+\frac{11e}{24}x^2+\cdots$

$\Rightarrow y-e+\frac{ex}{2}$

$=x^2\left[\frac{11e}{24}+\text{terms containing x and its higher powers}\right]$

$Lt_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e+\frac{ex}{2}}{x^2}$

$=Lt_{x\to 0}\left[\frac{11e}{24}+\text{terms containing x and its higher powers}\right]$

$=\frac{11e}{24}+0-\frac{11e}{24}$

Sol.14

Given that

$$f(x) = \begin{cases} x^3 & x \le 2 \\ ax^2 + bx, & x > 2 \end{cases}$$

Now $f(x)$ is differentiable at $x = 2 \therefore Lt_{x\to 2-} \frac{f(x)-f(2)}{x-2} = Lt_{x\to 2+} \frac{f(x)-f(2)}{x-2} \Rightarrow Lt_{x\to 2-} \frac{x^3-8}{x-2} = Lt_{x\to 2+} \frac{ax^2+bx-8}{x-2} \Rightarrow Lt_{x\to 2-} \frac{(x-2)(x^2+2x+4)}{x-2} = Lt_{x\to 2+} \frac{ax^2+bx-8}{x-2}$

$\Rightarrow Lt_{x\to 2-}(x^2 + 2x + 4) = Lt_{h\to 0} \frac{a(2+h)^2+b(2+h)-8}{2+h-2} \Rightarrow 4 + 4 + 4 = Lt_{h\to 0} \frac{4a+2b-8}{h} + 4a + b$

$\Rightarrow 12 = Lt_{h\to 0} \frac{4a+2b-8}{h} + 4a + b$

$\Rightarrow 4a + 2b = 8$ (i)

And $4a + b = 12$ (ii)

Subtracting (ii) from (i), we get $b = -4$

From (ii) $4a - a = 12 \Rightarrow a = 4 \therefore$ we have $a = 4, b = -4$

Sol.15

Given set is {1, 2, 3, …………..10}

Number of digits to be chosen =3

$\therefore$ Total number of ways ${}^{10}c_3 = \frac{10\times 9\times 8}{1\times 2\times 3} = 120$

If minimum number is 3, then remaining 2 digits can be selected from 7 digits 4, 5, 6, 7, 8, 9, 10 in ${}^{7}c_2 = \frac{7\times 6}{1\times 2} = 21\ ways$

If maximum number is 7, then remaining 2 digits can be selected from 6 digits 1, 2, 3, 4, 5, 6 in ${}^{6}c_2 = \frac{6\times 5}{1\times 2} = 15\ ways$

In the above cases a number in which minimum is 3 and maximum is 7, the third number will be out of 4, 5, 6. This number can be selected in ${}^{3}c_1 = 3$ ways

$\therefore$ Number of ways of selecting 3 digits out of {1, 2, 3, ………..10} such that either minimum is 3 or maximum is 7 are 21 + 15 – 3 = 33

$\therefore$ Number of favourable ways = 33

$\therefore$ Required probability $= \frac{33}{120} = \frac{11}{40}$

Sol.16

$T_n = \frac{1+a+a^2+\cdots+a^{n-1}}{n!} = \frac{\left(\frac{a^{n-1}}{a-1}\right)}{n!} = \frac{1}{(a-1)}\left[\frac{a^{n-1}}{n!}\right]$

$T_n = \frac{1}{(a-1)}\left[\frac{a^n}{n!} - \frac{1}{n!}\right]$ Put n$n = 1, 2, 3, \dots\dots\dots\dots$

$T_1 = \frac{1}{(a-1)}\left[\frac{a}{1!} - \frac{1}{1!}\right]$

$T_2 = \frac{1}{(a-1)}\left[\frac{a^2}{2!} - \frac{1}{2!}\right]$

$T_3 = \frac{1}{(a-1)}\left[\frac{a^3}{3!} - \frac{1}{3!}\right]$

………………… $\therefore T_1 + T_2 + T_3 + \dots\dots\dots\dots\dots\dots\dots$

$= \frac{1}{(a-1)}\left[\left(\frac{a}{1!} + \frac{a^2}{2!} + \frac{a^3}{3!}\right) - \left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right)\right] = \frac{1}{(a-1)}[(e^a - 1) - (e-1)] = \frac{1}{(a-1)}(e^a - e)$

Sol.17

Let $\Delta = \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$ (i)

Pitting $a + b = 0$ or $a = -b$ in $(i) we$ get

$\Delta = \begin{vmatrix} 2b & 0 & c-b \\ 0 & -2b & b+c \\ c-b & c+b & -2c \end{vmatrix}$

$= 2b(-b^2 - c^2 + 2bc) + (c-b)(2bc - 2b^2) = 0$ $\therefore (a+b)$ is factor of Δ Similarly $(b+c), (c+a)$ are factors of Δ

Let $\Delta = k(a+b)(b+c)(c+a)$ (ii)

$\therefore \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = k(a+b)(b+c)(c+a)$

Put $a = 0, b = 1, c = 1$

$\therefore \begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -2 \end{vmatrix} = k(0+1)(1+1)(1+0)$

$\Rightarrow 4 + 4 = 2k \Rightarrow k = 4$

$\therefore$from (ii), Δ=4$(a+b)(b+c)(c+a)$

Sol.18

Here $f(x) = [x], g(x) = |x|$

$D_f = R, R_f = I, D_g = R, R_g = (0, \infty)$

$\therefore R_f \subset D_g \therefore gof$ is defined.

Again $R_g \subset D_f$ $\therefore gof$ is defined.

Now $(fog)\left(-\frac{3}{2}\right) + (gof)\left(\frac{4}{3}\right) = f\left(g\left(-\frac{3}{2}\right)\right) + g\left(f\left(\frac{4}{3}\right)\right) = \left(\left|-\frac{3}{2}\right|\right) + g\left(f\left[\frac{4}{3}\right]\right) = f\left(\frac{3}{2}\right) + g(1)$

$= \left[\frac{3}{2}\right] + |1| = 1 + 1 = 2$

Sol.19

Put $x^2 = \cos\theta$

Given $\tan^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right] = tan^{-1}\left[\frac{\sqrt{1+cos\,\theta}+\sqrt{1-cos\,\theta}}{\sqrt{1+cos\,\theta}-\sqrt{1-cos\,\theta}}\right] = tan^{-1}\left[\frac{\sqrt{2cos^2\frac{\theta}{2}}+\sqrt{2sin^2\frac{\theta}{2}}}{\sqrt{2cos^2\frac{\theta}{2}}-\sqrt{2sin^2\frac{\theta}{2}}}\right]$

$= tan^{-1}\left[\frac{\sqrt{2}\,cos\frac{\theta}{2}+\sqrt{2}\,sin\frac{\theta}{2}}{\sqrt{2}\,cos\frac{\theta}{2}-\sqrt{2}\,sin\frac{\theta}{2}}\right] = tan^{-1}\left[\frac{cos\frac{\theta}{2}+sin\frac{\theta}{2}}{cos\frac{\theta}{2}-sin\frac{\theta}{2}}\right] = tan^{-}\left[\frac{1+tan\frac{\theta}{2}}{1-tan\frac{\theta}{2}}\right] = tan^{-1}\left[tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$

$= \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2}cos^{-1}x^2$

Sol.20

$Lt_{x\to 5-}f(x) = Lt_{x\to 5-}\frac{x-5}{|x-5|} + a = Lt_{x\to 5-}\frac{x-5}{-(x-5)} + a = -1 + a$

$Lt_{x\to 5+}f(x) = Lt_{x\to 5+}\frac{x-5}{|x-5|} + b = Lt_{x\to 5+}\frac{x-5}{x-5} + b = 1 + b$

Also $f(5) = a + b$ Since $f(x)$ is continuous at $x = 5$

$\therefore Lt_{x\to 5-}f(x) = Lt_{x\to 5+}f(x) = f(5) \Rightarrow -1 + a = 1 + b = a + b$

Taking first and third term, we get $b = -1$ Taking second and third term, we get

$1 + b = a + b \Rightarrow a = 1 \therefore a = 1, b = -1$

Sol.21

Given $y = \cos^{1}\left(\frac{3\cos x - 4\sin x}{5}\right)$ (i)

Put $\frac{3}{5} = r\ cos\ \theta$ and $\frac{4}{5} = r \sin\theta$

Squaring and adding, we get $r^2 = 1 \Rightarrow r = 1$

Dividing, we get $\tan\theta = \frac{4}{3}$

From (i) we get

$y = cos^{-1}(r\ cos\theta\ cos\ x - r\ sin\ \theta\ sin\ x)$

$= cos^{-1}(cos\ \theta\ cos\ x - sin\ \theta\ sin\ x)$

$\Rightarrow y = cos^{-1}[cos(x+\theta)]$

$\Rightarrow y = x + \theta$

$\Rightarrow y = x + tan^{-1}\frac{4}{3}$

$\Rightarrow \frac{dy}{dx} = 1$

Sol.22

$f(x) = x^3 + \frac{1}{x^3}$

$f'(x) = 3x^2 - \frac{3}{x^4} = \frac{3(x^6-1)}{x^4}$

$= \frac{3}{x^4}[(x^2)^3 - (1)^3]$

$= \frac{3}{x^4}[(x^2-1)(x^4+x^2+1)]$

$f'(x) = \frac{3(x^4+x^2+1)(x^2-1)}{x^4}$

For $f(x)$ to be strictly decreasing $f'(x) < 0$

$\Rightarrow \frac{3(x^4+x^2+1)(x^2-1)}{x^4} < 0$

$\Rightarrow x^2 - 1 < 0 \left[\therefore \frac{3(x^4+x^2+1)}{x^4} > 0\right] \Rightarrow x^2 < 1 \Rightarrow |x|^2 < 1$

$\Rightarrow |x| < 1$

$\Rightarrow -1 < x < 1$

Sol.23

Let$I = \int \frac{dx}{(x^2-1)\sqrt{x-1}}$

Put $\sqrt{x-1} = y \; or \; x-1 = y^2 \; or \; x = y^2+1$

$\therefore dx = 2ydy$

$I = \int \frac{2y\,dy}{|(y^2+1)^2-1|y} = 2\int \frac{dy}{y^4+2y^2}$

$\Rightarrow I = 2\int \frac{1}{y^2(y^2+2)} dy$

$\Rightarrow I = \int \left(\frac{1}{y^2} - \frac{1}{y^2+2}\right) dy$

$= \frac{y^{-1}}{-1} - \frac{1}{\sqrt{2}} tan^{-1}\frac{y}{\sqrt{2}} + c$

$= -\frac{1}{y} - \frac{1}{\sqrt{2}} tan^{-1}\frac{y}{\sqrt{2}} + c$

$\Rightarrow I = -\frac{1}{\sqrt{x-1}} - \frac{1}{\sqrt{2}} tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right) + c$

Sol.24

Let $I = \int_0^{\pi/2} \frac{x\,sin\,x\,cos\,x}{sin^4 x+cos^4 x} dx = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)} dx = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)\sin x\cos x}{\sin^x x+\cos^4 x} dx$

$$I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x} dx - I \Rightarrow 2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x} dx \Rightarrow I = \frac{\pi}{4}\int_0^{\pi/2} \frac{\sin x\cos x}{\sin^4 x+\cos^4 x} dx$$

Put $sin^2 x = t \Rightarrow 2\sin x\cos x\, dx = dt \Rightarrow \sin x\cos x\, dx \frac{dt}{2}$

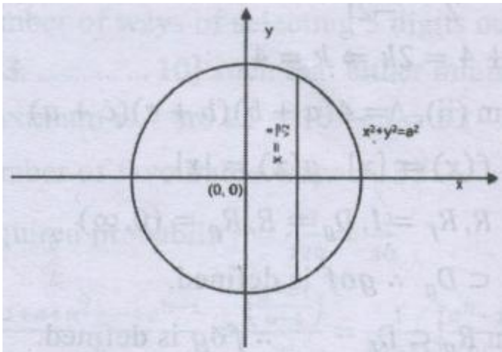
When $x = 0, t = sin^2\theta = 0$When $x = \frac{\pi}{2}, t = sin^2\frac{\pi}{2} = 1$

$$I = \frac{\pi}{8}\int_0^1 \frac{1}{t^2+(1-t)^2} dt = \frac{\pi}{8}\int_0^1 \frac{1}{2t^2-2t+1} = \frac{\pi}{16}\int_0^1 \frac{1}{t^2-t+\frac{1}{2}} dt = \frac{\pi}{16}\int_0^1 \frac{1}{\left(t-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} dt$$

$= \frac{\pi}{16}\frac{1}{\left(\frac{1}{2}\right)}\left[tan^{-1}\left(\frac{t-\frac{1}{2}}{\frac{1}{2}}\right)\right]_0^1 = \frac{\pi}{8}[tan^{-1}(2t-1)]_0^1 = \frac{\pi}{8}[\tan^{-1} -1\; tan^{-1}(-1)]$

$= \frac{\pi}{8}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi}{8} \times \frac{\pi}{2} = \frac{\pi^2}{16}$

Sol.25



$-\sqrt{a^2-x^2} \leq y \leq \sqrt{a^2-x^2}$

$\frac{a}{\sqrt{2}} \leq x \leq a$

Area $= \int_{\frac{a}{\sqrt{2}}}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx$

$= 2\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2-x^2 dx}$

$= \left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$

$= \frac{\pi a^2}{2} - \frac{a^2}{2} - \frac{\pi a^2}{4} = \frac{\pi a^2}{4} - \frac{a^2}{2}$

$= \frac{a^2}{4}(\pi - 2) sq\ units$

Sol.26

Given equation is $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$, $\frac{dy}{dx} = \frac{1+e^{\frac{x}{y}}}{e^{\frac{x}{y}}\left(\frac{x}{y}-1\right)}$, Put $x = vy\ or\ \frac{dx}{dy} = v + y\frac{dv}{dy}$ From

(i), we get

$\frac{1}{v+y\frac{dv}{dy}} = \frac{1+e^v}{e^v(v-1)} \Rightarrow v + y\ \frac{dv}{dy} = \frac{e^v(v-1)}{1+e^v} \Rightarrow y\frac{dv}{dy} = -\frac{e^v+v}{1+e^v} \Rightarrow \frac{1+e^v}{v+e^v}dv = -\frac{dy}{y}$

Intergrating both sides, we get

Log $|v+e^v| = -$log$|y|$ +log$|c|$

log$|v+e^v|$ =log$\left|\frac{c}{y}\right|$ or $(v+e^v) = \frac{c}{y}$

$\Rightarrow \left(\frac{x}{y} + e^{\frac{e}{y}}\right)y = c = x + ye^{\frac{x}{y}} = c$

Sol.27

$\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$

$\therefore |\vec{a}| = \sqrt{4+1+1} = \sqrt{6},$

$|\vec{b}| = \sqrt{9+16+1} = \sqrt{26}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$$

$= (1-4)\hat{\imath} - (-2-3)\hat{\jmath} + (8+3)\hat{k} = 3\hat{\imath} + 5\hat{\jmath} + 11\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{9+25+121} = \sqrt{155}$

Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{1}{\sqrt{155}}(-3\hat{\imath} + 5\hat{\jmath} + 11\hat{k})$

Sol.28

The line passes through $A(3,4,1)$ and $B(5,1,6)$

$\therefore \vec{a} = 3\hat{\imath} + 4\hat{\jmath} + \hat{k}, \vec{b} = 5\hat{\imath} + \hat{\jmath} + 6\hat{k}$

$\vec{b} - \vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

Let $\vec{r}$ be position vector of any point on the given line.

$\therefore \vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$\vec{r} = (3\hat{\imath} + 4\hat{\jmath} + \hat{k}) + \lambda(2\imath - 3\hat{\jmath} + 5\hat{k})$ (i)

Let $P(x, y, 0)$ be the point where the line AB crosses the xy plane. Now P lies on (i)

$\therefore x\hat{\imath} + y\hat{\jmath}$ lies on (i)

$\therefore x\hat{\imath} + y\hat{\jmath} = (3\hat{\imath} + 4\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} - 3\hat{\jmath} + 5\hat{k})$

$\therefore x = 3 + 2\lambda$ (ii)

$y = 4 - 3\lambda$ (iii)

$0 = 1 + 5\lambda$ (iv)

From (iv), $\lambda = -\frac{1}{5}$ From (ii), $x = 3 - \frac{2}{5} = \frac{13}{5}$ From (iii)$y = 4 + \frac{3}{5} = \frac{23}{5}$

Point is $(\frac{13}{5}, \frac{23}{5}, 0)$

Sol.29

Here $P(A) = 0.38$ and $P(A \cup B) = 0.69$. Let $P(B) = p$

Since A and B are independent

$\therefore P(A \cap B) = P(A) \times P(B)$

$P(A \cap B) = (0.38)p$ Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.69 = 0.38 + p - (0.38$

$0.31 = 0.62p \Rightarrow p = \frac{1}{2}$

$\therefore P(B) = \frac{1}{2} = 0.5$

Sol.30

Let the shooter fire n times.

Now $q = \frac{1}{4}, p = \frac{3}{4}$

$P(X = x) = {}^{n}c_x \left(\frac{3}{4}\right)^x \left(\frac{1}{4}\right)^{n-x} = {}^{n}c_x \frac{3x}{4^n}$

Now $P(hitting\ the\ target\ at\ least\ once) > 0.99$

$\therefore P(x \geq 1) > 0.99$

$\Rightarrow 1 - P(x = 0) > 0.99$

$\Rightarrow 1 - {}^{n}c_0 \frac{1}{4^n} > .99$

$\Rightarrow \frac{1}{4^n} < 0.01$

$\Rightarrow 4^n > \frac{1}{0.01}$

$\Rightarrow 4^n > 100$ (i)

$\therefore$ Minimum value of n satisfying (i) is 4

$\therefore$ The shooter must fire 4 times